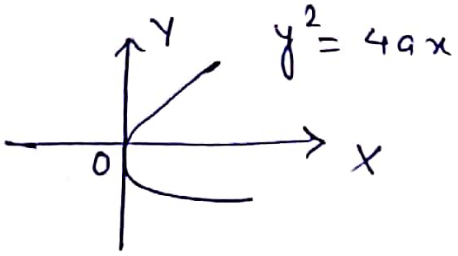
Curve Tracing (Cartesian Curves)

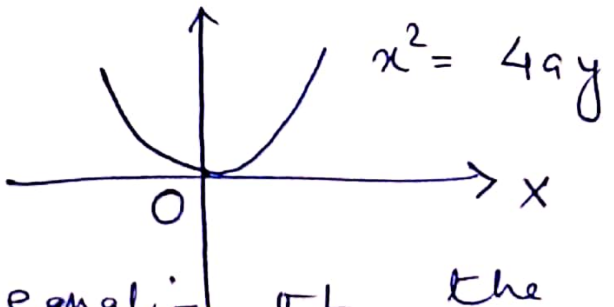
Rules for tracing Cartesian Curves.

(I) Find the symmetry of the curve about any line.

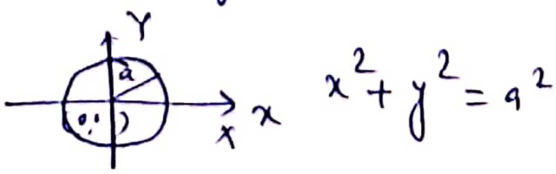
(i) A curve is symmetrical about x-axis if its equation has only even powers of y (i.e if by putting $-y$ for y the equation remains unchanged) e.g, $y^2 = 4ax$ is symmetrical about x axis.



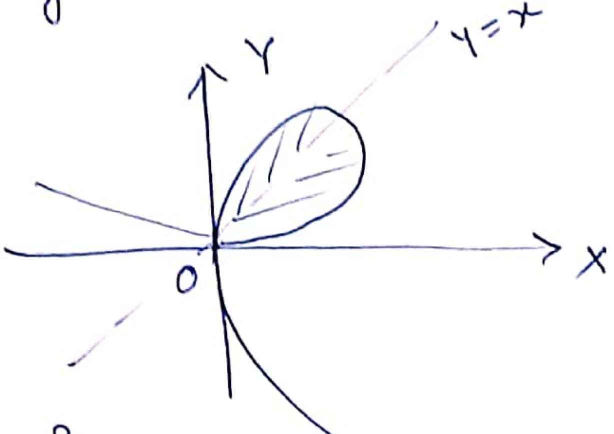
(ii) A curve is symmetrical about y axis if its equation contains only even powers of x e.g, $x^2 = 4ay$ is symmetrical about y axis.



(iii) If the equation of the curve contains only even powers of x and y, it is symmetrical about both axes, e.g. $x^2+y^2 = a^2$ is symmetrical about both axes.



(iv) If on interchanging x and y the equation of the curve remains unchanged, it is symmetrical about the line $y = x$, e.g. $x^3 + y^3 = 3axy$ is symmetrical about the line $y = x$.



$x^3 + y^3 = 3axy$ ~~is~~ called (Folium of Descartes).

This curve is symmetrical about the line $y = x$.

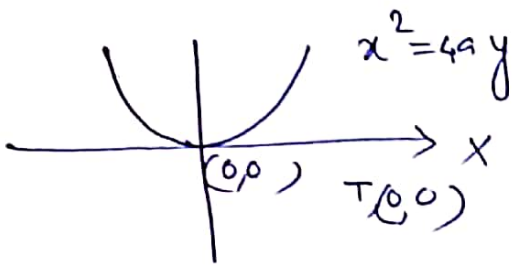
II Find whether the curve passes through the origin

For this put $x = 0$, $y = 0$ in the equation of the curve, if the equation is satisfied, the curve passes through the origin, e.g $x^2 = 4ay$ passes through the origin.

III Find the equation of the tangents at origin

If the curve passes through the origin, find the equation of the tangents at the origin. For this equating the lowest degree term in the equation to zero. e.g, in the curve $x^2 = 4ay$, the lowest degree term is $4ay$, equating it to zero, we get $y = 0$

i.e, x axis is the tangent at origin.



Here x-axis is tangent to the given curve.

(IV) Find the point of intersection of the curve and the axes.

Now we should see at what points the curve cuts the x-axis and y axis. In order to determine the co-ordinates of the point of intersection with the x-axis we shall put $y=0$ in the equation of the curve because the y-co-ordinate of any point situated on the x axis $=0$.

Similarly, in order to find out the co-ordinates of the point of intersection with the y axis we shall put $x=0$ in the equation of the curve.

(V) Find the regions of the curve

For this, solve the equation for y. Find such values of x for which y becomes imaginary. For example, if the curve is

$$y^2(2a-x)=x^3 \text{ then } y^2=\frac{x^3}{(2a-x)}$$

If $x<0$, y is imaginary and if $x>2a$, y is again imaginary. So no part of the curve is either in the left of the origin or in the right of $x=2a$.

(VI) Find the asymptote

(i) In order to obtain the asymptote parallel to the axis of x, equate to zero the Co-efficient of the highest power of x. For example, if the Curve be of the nth degree and the term Containing x^n be absent, then the Co-efficient of x^{n-1} equated to zero will give the asymptote ∥ to the axis of x.

(ii) If both the terms Containing x^n and x^{n-1} be absent, then the Co-efficient of x^{n-2} equated to zero will give asymptote Parallel to the axis of x.

(iii) To get asymptote Parallel to the axis of y equate to ~~to~~ zero the Co-efficient of the highest power of y. For example, if the Curve be of the nth degree and the term Containing y^n be absent, then the Co-efficient of y^{n-1} equated to zero will give the asymptote parallel to the axis of y.

(iv) If the equation of the curve be of nth degree and the Co-efficient of x^n is not zero then there will be no asymptote parallel to x-axis, Similarly, if the Co-efficient of y^n is not zero then there will be no asymptote parallel to y axis.

For example $x^3+y^3 = 3axy$ will have no asymptote parallel either to x axis or y axis. Above curve have an oblique Asymptote'